The Industrial Path to a Dyson Swarm

A Measurement and Growth Framework for Self-Expanding Space Industry

Solar Dyson Initiative

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Abstract

A practical Dyson swarm is not a single artifact. It is the large-scale consequence of an extraterrestrial industrial system able to maintain and expand its own productive capacity. This paper proposes a compact program framework for reasoning about that transition. Four headline diagnostic families describe the system's condition: accepted material throughput, delivered power, industrial closure, and net installed productive-capacity growth. The framework treats closure as a structured record of quantitative supply fractions and categorical acceptance gates, not as one mass percentage, and defines productive capacity against a stated reference industrial basket. A constrained-growth model then connects in-boundary supply, bounded imports, reinvestment, commissioning delay, retirement, bootstrap leverage, and net growth. Its central result is conditional: at a stated positive fractional-growth target, a recurring new-Earth-dependent input requirement per unit capacity that remains bounded away from zero, together with finite inventory and a bounded Earth channel, imposes a finite ceiling on industrial scale. Closure therefore governs whether sustained fractional growth can decouple from Earth, even when power or material throughput is the immediate bottleneck. The paper uses this result to motivate a four-regime dependency model, compare orbital swarms with radiation-supported bubble elements, and specify measurements that could falsify or refine the empirical assumptions. It is a framework for technical argument, not a forecast date or a claim that stellar-scale construction is presently feasible.

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Status of this document. This is a technical concept paper. It defines a reporting framework, derives a reduced growth model, and makes its assumptions explicit so that they can be challenged. Numerical examples are illustrative parameter sets, not cost estimates, schedules, forecasts, or investment claims. The physical, economic, political, and legal feasibility of any particular deployment remains an open engineering question.

The accompanying public summary is available at solardysoninitiative.com. Correspondence: contact@solardysoninitiative.com.

1 Thesis and scope

1.1 The claim

The phrase *Dyson sphere* encourages the wrong project decomposition. It suggests one object, one design, and eventually one completion event. A physically useful interpretation is instead a population of independently deployed systems: power collectors, mines, refineries, factories, transport vehicles, communications relays, radiators, habitats, observatories, and control infrastructure. Most elements move in solar orbit. Some may use radiation pressure or active thrust to occupy non-Keplerian positions. They need not share a design, owner, purpose, orbit, or date of construction.

Freeman Dyson's original proposal was observational: a technically advanced civilization converting starlight at large scale should return energy to its environment as lower-temperature radiation, making infrared searches relevant (Dyson 1960). Later technical literature has separated a rigid shell from orbiting or radiation-supported populations and has emphasized the shell's mechanical and dynamical problems (Wright 2020). This paper begins one level earlier than either observational signature or collector geometry. It asks what industrial condition would make sustained, large-scale deployment possible.

The central claim is:

> A Dyson swarm is not usefully treated as a construction project. It is the asymptotic output of a space industry whose maintenance and expansion can become progressively less dependent on Earth.

This claim changes the governing questions. The leading question is not "How many collectors have launched?" It is "Can the system make the next increment of productive capacity without requiring an equal or larger increment from Earth?" The answer depends on material throughput, power delivered to productive loads, industrial closure across essential inputs, and net growth after maintenance and retirement.

These four headline records are written

$$\mathbf{z}(t) = [\dot{m}(t),\ P(t),\ \mathbf{C}(t),\ g(t)]\,.$$

They are **four headline diagnostic families**, not a complete mathematical state. Their minimality is a design hypothesis to test, not a theorem. The full system includes inventories, process-specific capacities, transport state, reliability, orbital occupancy, information dependencies, and many other variables. The point of the dashboard is narrower: together the four records prevent four common category errors.

- Raw material movement is not accepted industrial output.
- Nameplate generation is not power available at a load.
- Local mass fraction is not functional independence.
- Gross commissioning is not net growth.

1.2 What this paper does not claim

This paper does not establish that a self-expanding extraterrestrial industry is near-term, profitable, safe, politically acceptable, or inevitable. It does not select the Moon, asteroids, or Mercury as an exclusive resource base. It does not require one universal self-replicating machine. It does not assume that human settlement is necessary for robotic industrial expansion, or that robotic expansion is desirable without human settlement. It does not predict a date at which any regime transition occurs.

The model is deliberately technology-agnostic within broad physical constraints. It assumes no violation of known mechanics, thermodynamics, or electromagnetism. It permits improvements in autonomy, manufacturing, materials, propulsion, power conversion, and computation, but it does not insert an unspecified breakthrough as a free variable. When a result depends on a constant yield, delay, failure rate, or input intensity, that dependency is stated.

The framework also distinguishes **possibility** from **program status**. A laboratory process that extracts oxygen from simulated regolith is evidence about a process. It is not kilograms per day of accepted output at an operating site. A robot that prints one structural part is evidence about fabrication. It is not closure of the machine tool, controls, feedstock, inspection, and maintenance chain. A solar array deployed in orbit is generation capacity. It is not delivered power at a distant refinery after storage, transmission, conversion, and availability losses.

1.3 Criteria for a useful program model

A variable belongs in a high-level program model if it satisfies five tests.

1. **It is physically measurable.** The quantity can be tied to mass, energy, qualified output, or installed productive service rather than rhetoric.
2. **It changes when the system becomes more capable.** A real improvement should move at least one reported quantity in the expected direction.
3. **It resists obvious gaming.** Moving unprocessed regolith twice should not double productive throughput. Pre-positioning imported spares should not permanently raise closure.
4. **It supports comparison across architectures.** A lunar surface factory and an asteroid-processing fleet should be comparable at the level of output and dependency even when their machinery differs.
5. **It exposes bottlenecks.** The reporting scheme should make a missing low-mass input or a negative replacement balance visible.

No four scalars satisfy these tests perfectly. The proposed dashboard works only when each headline value is accompanied by a defined boundary and a small set of disaggregated measurements. That reporting discipline is part of the model.

2 From stellar collector to industrial population

2.1 The physical reference scale

The nominal solar luminosity adopted by the International Astronomical Union is

$$L_{\odot}^{\mathrm{N}} = 3.828 \times 10^{26}\ \mathrm{W}$$

(Prša et al. 2016). This is a physical reference scale, not a market size and not an initial design requirement. A collector of projected area A at distance r from the Sun intercepts, before conversion losses,

$$P_{\mathrm{incident}} = \frac{L_{\odot} A}{4\pi r^2}.$$

For an optically thin population whose units do not significantly shadow one another, total intercepted power is approximately the sum over units. At higher coverage fractions, mutual shadowing, reradiation, orbital geometry, and radiative feedback require a coupled treatment (Wright 2020). Nothing about this relation requires a monolithic surface. Every additional collector can be an independent increment.

That incrementality is the architectural advantage. A useful first collector can precede the millionth by centuries. Units can be replaced without redesigning the population. Different temperature regimes and power users can occupy different orbits. Mechanical failures need not transfer load through one structure, although trajectory and debris effects can propagate. The same qualities also create coordination problems: conjunctions, beam allocation, spectrum use, debris, authentication, and shared resource rules become system infrastructure rather than afterthoughts.

2.2 The system boundary

"Made in space" is not a sufficiently precise boundary. A rigorous report must state at least:

- the geographic boundary, such as Earth surface, cislunar space, or all nonterrestrial sites;
- the accounting window $[t_0, t_1]$;
- which inventories existed at t_0;
- which energy, material, information, labor, and transport flows cross the boundary;
- which output acceptance tests apply; and
- the reference technology basket used to compare productive capacity.

Source provenance and processing location are separate attributes. Every physical flow is assigned to one of four mutually exclusive classes by a boundary-entry and use-cycle rule. **New Earth import** means first use inside the declared boundary after crossing from Earth, regardless of recycled content or processing performed on Earth. **Recycled Earth-origin** means Earth-origin material recovered after a previous use inside that boundary. **Virgin nonterrestrial** means first use after extraction from a nonterrestrial source, and **recycled nonterrestrial-origin** means its recovery after a previous in-boundary use. Boundary entry takes precedence: steel recycled on Earth and then launched is a new Earth import; only a later in-space recovery moves it to recycled Earth-origin. A product processed outside Earth may contain a mixture, whose fractions propagate recursively through yields, scrap, and recycling. Software, remote labor, design authority, and cryptographic credentials are tracked as categorical service dependencies rather than fictitious mass flows.

Trade can remain rational at very high closure. The objective is not autarky for its own sake. The relevant question is whether Earth support is a chosen exchange or a scaling requirement. A system that imports one kilogram of specialized electronics per year may be operationally robust. If every added factory requires another kilogram per year and the import channel is capped, the same dependency eventually imposes a ceiling.

2.3 Productive capacity and the reference basket

Growth requires a stock variable, but installed capital and current availability must not be conflated. Let k_i^{inst} be accepted installed capacity of process i in its natural unit: kilograms of refined aluminum per day, square meters of qualified photovoltaic film per day, tug-tonne-kilometers per year, controller boards per month, or maintenance work orders closed per week. Let k_i^{avail} be the service currently available after temporary outage, derating, missing inputs, and duty-cycle limits. Both vectors are primary records.

For high-level comparison, define one **reference industrial complex-equivalent** (CE) as a fixed basket of productive services and duty cycles. If b_i is the process-i capacity required by that basket, then a bottleneck projection is

$$K_{\text{inst}} = \min_i \left(\frac{k_i^{\text{inst}}}{b_i} \right), \qquad K_{\text{avail}} = \min_i \left(\frac{k_i^{\text{avail}}}{b_i} \right).$$

K_{inst} and K_{avail} are measured in complex-equivalents. The minimum is a convenient projection, not automatically a conservative attribution: it is valid only when capacity can be allocated to the basket without co-location conflicts, shared-capacity double counting, or incompatible duty cycles. Ten refineries do not constitute ten complete complexes if transport or controls support only three, but a final nonterrestrial-origin bottleneck component also must not magically convert stranded Earth-origin equipment into a wholly nonterrestrial CE. Where processes substitute, share capacity, or span a network, compute CE through an explicit allocation or network-flow optimization and retain source attribution across the entire basket. Any reported value must publish its basket, allocation assumptions, binding constraints, and version. When technology changes the basket, the time series must be rebased.

For compact derivations below, $K \equiv K_{\text{inst}}$. Temporary repair or reactivation can raise K_{avail} but never counts as new K_{inst}. New accepted equipment, accepted capability upgrades, permanent derating, and retirement change K_{inst}. The scalar projection is intentionally imperfect: it gives g and bootstrap leverage a common unit, but it can hide the underlying vectors. Every capacity report therefore publishes both installed and available CE, the process vectors, and the binding allocation constraint.

2.4 A population, not an object

The industrial interpretation makes "completion" a category error. The population grows while additional useful deployment exceeds retirement. It can pause, specialize, relocate, or shrink. Its composition follows demand. A mature system could allocate output among power collection, computation, habitats, transportation, science, reserves, restoration, or purposes not presently anticipated. Full interception of solar luminosity is neither required nor necessarily optimal.

The immediate program objective is therefore not a collector count. It is the ability to produce, operate, repair, and commission useful systems under progressively weaker dependence on terrestrial launch. A swarm is the visible capital stock produced by that capability.

3 The four-diagnostic dashboard

3.1 Why these four

The proposed observables cover four different questions:

Observable	Unit	Program question	Failure hidden by a naive proxy
$\dot{m}$	kg/time	Is accepted physical output flowing?	Raw excavation without qualified products
P	W	Can productive loads receive usable energy?	Nameplate generation without delivery or availability
$\mathbf{C}$	fractions plus gates	Which quantitative inputs and categorical services close?	High mass closure with a critical imported component or authority
g	1/time	Is complete productive capacity increasing after losses?	Gross construction that only replaces retirement

The four families are jointly useful and individually insufficient. High throughput without closure can describe an imported factory processing in-space bulk material. High closure without throughput can describe a beautifully closed laboratory apparatus with negligible output. High delivered power without productive capacity can describe an idle grid. Positive growth supported by imports that rise proportionally with K can describe rapid deployment, but not self-expansion.

3.2 Material throughput $\dot{m}$

Material throughput is a stage vector before it is a headline scalar. For stage s,

$$\dot{m}_s = \frac{\Delta m_{s,\text{accepted}}}{\Delta t},$$

with units of mass per time. At minimum, report:

1. extracted feedstock;
2. beneficiated or concentrated material;
3. refined material meeting chemistry specifications;
4. qualified fabrication feedstock;
5. accepted fabricated output embodied in usable products;
6. recovered recycled material;
7. rejected and scrapped material.

The same kilogram must not be added across stages. A headline $\dot{m}$ should mean the accepted material embodied in products crossing a declared reporting boundary. Raw regolith moved, tailings processed, rework, and internal transfers remain operational metrics but do not inflate final throughput.

This definition exposes yield. If $\dot{m}_{\text{extract}}$ is large while $\dot{m}_{\text{accepted}}$ is small, the program has an extraction statistic, not an industrial material flow. Reporting each stage also prevents a high-volume commodity such as shielding aggregate from concealing zero output in silicon, bearings, optics, or conductors.

Mass is not value and is not complexity. The role of $\dot{m}$ is to measure physical scale. Closure and the process-capacity vector carry the information that tonnage cannot.

3.3 Delivered power P

Power is reported by a meter at a named load, not inferred by subtracting vaguely defined losses from nameplate generation. For load L, let $P_L(t)$ be usable power crossing the load boundary. In the following upstream reconciliation, generation, imports, discharge, and charge are metered at declared component boundaries; the explicit conversion and transmission losses carry those flows to the load meter. Allocated generation is net of generator auxiliary loads, and every meter location must be published so that no loss is counted twice. The reconciliation may be written

$$P_L = P_{\text{gen,allocated}} + P_{\text{discharge}} + P_{\text{import}} - P_{\text{charge}} - P_{\text{loss,conversion}} - P_{\text{loss,transmission}},$$

with storage state of charge reconciled over the reporting window. Storage conversion and self-discharge losses are assigned once - to the conversion-loss ledger or the state-of-charge reconciliation, not both. Curtailment remains upstream of allocated generation rather than masquerading as a downstream loss. The mean delivered power over a window is

$$\bar{P}_L = \frac{1}{\Delta t} \int_{t_0}^{t_1} P_L(t)\, dt.$$

A useful report also gives peak power, firm power at a stated availability, interruption distribution, voltage or beam quality, and thermal conditions. A megawatt intermittently available at a collector is not interchangeable with a megawatt of dispatchable electrical power at a furnace.

Space-based collection and remote transmission have a long technical lineage; Glaser's solar power satellite proposal coupled orbital collection to microwave delivery, while later microwave work treated source, apertures, propagation, pointing, receiver, conversion, and safety as one end-to-end link (Glaser 1968; Brown and Eves 1992). The same accounting principle applies inside an extraterrestrial industrial network. Generation, storage, beam or cable transmission, conversion, pointing, receiver availability, and waste heat all belong between nameplate power and productive service.

3.4 Industrial closure C

Industrial closure is a structured record of which replenishment and expansion requirements the system can satisfy inside a declared boundary, target, window, and stress case. Quantitative physical flows and categorical services are evaluated separately; authorization, software support, or recovery authority is not assigned an invented mass fraction.

For quantitative replenishment class j with required flow $R_j > 0$, let $L_{j,\text{actual}}$ be qualified in-boundary supply actually used. Let $\widehat{\mathbf{L}}_{\text{stress}}$ be one **jointly feasible** supply vector demonstrated under a declared disruption, dwell time, opportunity cost, shared-capacity allocation, and import cap. Its components must be achievable simultaneously; independent componentwise maxima from incompatible production schedules cannot be combined. Define realized and capability closure as

$$c_j^{\text{real}} = \min\left(1, \frac{L_{j,\text{actual}}}{R_j}\right), \qquad c_j^{\text{cap}} = \min\left(1, \frac{\widehat{L}_{j,\text{stress}}}{R_j}\right).$$

Classes with $R_j = 0$ in the stated target are excluded rather than assigned a perfect score. Realized closure records actual in-boundary sourcing; capability closure records resilience. Material origin is the separate recursive provenance record defined above. An intentionally idle fallback may therefore have low c_j^{real} but high c_j^{cap}. For categorical class h, define a preregistered acceptance predicate $A_h(\text{window}, \text{stress}) \in \{0, 1\}$. The primary closure record is

$$\mathbf{C} = \left[\mathbf{c}^{\text{real}},\ \mathbf{c}^{\text{cap}},\ \mathbf{A}\right].$$

Two summaries are useful if the vector remains available. Embodied mass closure is

$$C_m = \frac{M_{\text{NT,virgin}} + M_{\text{NT,recycled}}}{M_{\text{Earth,new}} + M_{\text{Earth,recycled}} + M_{\text{NT,virgin}} + M_{\text{NT,recycled}}},$$

and a conservative essential-input summary is

$$C_{\text{crit}}^{\text{cap}} = \min_{j \in \mathcal{E}} c_j^{\text{cap}},$$

for one declared output or campaign after recursive provenance attribution. The essential set $\mathcal{E}$ is tied to a versioned reference basket, declared non-substitutable cut sets, and preregistered substitution rules; otherwise teams could improve the score merely by redefining "essential." C_m measures nonterrestrial material provenance. $C_{\text{crit}}^{\text{cap}}$ exposes a potential quantitative stop condition, while the predicates expose categorical stop conditions. None replaces the bill of requirements.

Closure is target-relative. A plant may be closed for current maintenance at zero growth but not for commissioning another plant. It may be closed for one year using inventories but not for ten. It may fabricate a controller board locally while importing the semiconductor die, photolithography service, or design key. The paper develops these distinctions in Section 4.

3.5 Net capacity growth g

Let I_K^{new} be newly commissioned accepted installed capacity and I_K^{upgrade} be accepted incremental CE from permanent capability upgrades, both per unit time. Define $I_K = I_K^{\text{new}} + I_K^{\text{upgrade}}$. Let R_K be installed capacity permanently retired, destroyed, or derated. Temporary outages and subsequent repair change K_{avail}, not any of these installed-stock flows. The instantaneous net installed-capacity growth rate is

$$g(t) = \frac{1}{K(t)}\frac{dK}{dt} = \frac{I_K(t) - R_K(t)}{K(t)}.$$

g has units of inverse time. For an interval, a log growth estimate is

$$\bar{g}_{\log} = \frac{\ln[K(t_1)/K(t_0)]}{t_1 - t_0}.$$

The discrete compound annual growth rate, when the interval is measured in years, is instead

$$\mathrm{CAGR} = \left[\frac{K(t_1)}{K(t_0)}\right]^{1/(t_1-t_0)} - 1.$$

An operational flow approximation is

$$g_{\text{flow}} \approx \frac{\Delta K_{\text{new}} + \Delta K_{\text{upgrade}} - \Delta K_{\text{permanently retired}}}{\bar{K}\,\Delta t}.$$

The log and flow measures agree only over short windows or sufficiently steady trajectories; CAGR is the discrete annual transformation of the same endpoints. Reports should state which is used.

If g is a constant continuously compounded rate, doubling time is

$$t_2 = \frac{\ln 2}{g}.$$

If g_a is instead a discrete annual fractional rate, the exact doubling time is $\ln 2/\ln(1+g_a)$. This distinction is small at low rates and material at high ones.

Positive g is not sufficient evidence of self-expansion. Earth may be delivering most commissioned capacity. Source attribution must cover the full accepted CE, not merely the last locally made bottleneck component, and closure must be evaluated against the expansion target. Conversely, a system at $g = 0$ may have significant nonterrestrial commissioning if all output is replacing retirements. Gross reproduction and net growth answer different questions.

3.6 Why familiar alternatives are inadequate

Launch mass measures terrestrial logistics, not local industry. Collector count ignores size, power, and purpose. Nameplate megawatts ignore delivery. Cumulative mined tonnage ignores yield and current rate. Technology readiness levels describe component maturity, not integrated production. Capital expenditure becomes increasingly ambiguous when much capital is made internally. Calendar phases express intent but do not reveal whether the physical system crossed a regime boundary.

Those metrics remain useful. They simply do not replace the four questions: What qualified matter flows? What power reaches the work? Which dependencies are local? Is complete productive capacity growing after loss?

4 Industrial closure as the scaling constraint

4.1 From self-reproducing automaton to industrial ecology

Von Neumann's formal work established that self-reproduction can be described at the level of abstract automata and construction logic rather than as the literal copying of a familiar machine (von Neumann 1966). Moving from that mathematical result to a physical factory requires many additional assumptions. Space-manufacturing studies provide the narrower engineering motivation for a network whose tools and services reproduce functions without requiring one atom-for-atom universal copier (Freitas and Gilbreath 1982; Metzger et al. 2013; Ellery 2016).

That set is an **industrial ecology**. A mine feeds separators. Separators feed chemical and metallurgical processes. Those processes feed machine tools, deposition systems, and assembly. Transport connects sites. Power supports every stage. Metrology and qualification determine whether output is usable. Controls schedule the network. Maintenance returns failed equipment to service.

No one unit is self-sufficient. Closure belongs to the network.

NASA's *Advanced Automation for Space Missions* made this point concrete in a study of an automated, multiproduct lunar manufacturing facility. The report separated qualitative closure - whether every required substance could in principle be derived through the proposed process network - from quantitative closure - whether the required quantities could actually be produced - and examined the throughputs needed across that network (Freitas and Gilbreath 1982). The study is prior art, not a validation of modern feasibility. Its durable contribution is the accounting question: for every required function, input, and production rate, where does the replacement come from?

Later bootstrapping models similarly examined a sub-replicating lunar industry that would add capabilities over time rather than arrive fully closed (Metzger et al. 2013; Ellery 2016). Numerical assumptions and proposed tool chains remain open to challenge. The transferable modeling insight is that closure is approached through an evolving network, and early nonterrestrial bulk production can coexist with continuing imported "vitamins."

4.2 A requirements model

For each essential input or service class j, define the required flow for maintenance, expansion, and other demand:

$$R_j = a_j^M K + a_j^I I_K + d_j.$$

Here:

- a_j^M is maintenance input intensity, in units of input j per CE per unit time;
- a_j^I is incremental-capacity input intensity, in units of input j per accepted CE from new commissioning or permanent upgrade;
- $I_K = I_K^{\text{new}} + I_K^{\text{upgrade}}$ is the accepted incremental-capacity rate, in CE per unit time; and
- d_j is final demand for input j outside industrial maintenance and expansion.

For each quantitative class, let L_j be qualified supply produced inside the geographic boundary, V_j be qualified inflow from non-Earth sites outside that boundary, U_j be new supply crossing from Earth, and S_j be usable in-boundary inventory. Thus, for example, asteroid material delivered into a cislunar boundary is V_j, not L_j or U_j. Ignoring inventory losses for the moment and taking $\dot{S}_j > 0$ to mean stock accumulation, feasibility over a reporting interval requires

$$L_j + V_j + U_j - \dot{S}_j \geq R_j \qquad \text{for every essential } j.$$

The minimum new import consistent with the measured in-boundary flow is

$$U_{j,\text{required}} = \left[R_j - L_j - V_j + \dot{S}_j\right]_+.$$

For an Earth-versus-all-nonterrestrial boundary, every non-Earth supplier is inside and $V_j = 0$. Under exact balance, no inventory change, and $L_j < R_j$, the expression then reduces to $U_{j,\text{required}} = (1 - c_j^{\text{real}})R_j$. An inventory draw can keep a system operating while current supply is deficient, but it cannot be counted as current production or used indefinitely. A closure report therefore includes every external inflow, inventory change, losses, and stockpiling before the measurement window.

Quantitative classes may be physical materials, finished components, transport service, test capacity, or maintenance labor measured as flows. Their units need not match because each constraint is evaluated separately. Categorical dependencies such as an indispensable software entitlement, signing authority, or legal authorization are instead tested as predicates A_h; they never enter the ratio or inventory algebra. A weighted average is useful only as a summary after every quantitative constraint and categorical gate has been tested.

4.3 Recursive attribution

Local assembly is not the same as nonterrestrial provenance. If a board is assembled in space from imported semiconductor devices, its embodied record carries those imports upstream. If a refinery uses a locally fabricated pressure vessel but an imported catalyst, both dependencies appear in the product's requirements. Material-flow analysis supplies the discipline of explicit boundaries, stocks, balanced flows, and allocation rules (Brunner and Rechberger 2016). This framework's four-way convention then requires the new Earth, recycled Earth-origin, virgin nonterrestrial, and recycled nonterrestrial-origin fractions of each output to sum to one after process yield and scrap are reconciled.

Let $\mathbf{x}$ be in-boundary gross output by process or commodity, $\mathbf{u}$ aggregate all boundary-crossing external inflows (with $\mathbf{V}$ and $\mathbf{U}$ retained as separate subledgers), $\mathbf{y}$ final maintenance and expansion demand, and A a direct-requirements matrix. A simple input-output feasibility condition is

$$\mathbf{x} + \mathbf{u} \geq A\mathbf{x} + \mathbf{y}.$$

This borrows the production-network logic of input-output analysis (Leontief 1936). The inequality is componentwise. Where $A \geq 0$, the fixed-coefficient approximation is reasonable, and its spectral radius satisfies $\rho(A) < 1$, the nonnegative Leontief inverse $(I - A)^{-1}$ exposes direct and embodied upstream requirements. Where those conditions fail, or substitution, joint production, batch effects, shared equipment, or nonlinear yields matter, a process model or optimization program must replace the fixed matrix. The framework does not assume that one static Leontief matrix accurately describes centuries of technical change. It uses the matrix to prevent an accounting shortcut: intermediate products do not erase their imported antecedents.

Recycling receives the same treatment. Recovered metal reduces virgin demand but requires collection, sorting, energy, processing, and yield. Counting both recovered feed and the product containing it as new mass double-counts the same material. Earth-origin atoms remain Earth-origin after in-space recycling; nonterrestrial-origin atoms remain nonterrestrial-origin. A consistent material balance reports both origin classes, virgin versus recovered input, losses, inventory change, and accepted output for one declared product or campaign.

4.4 Mass closure and functional closure

Suppose a ten-tonne excavation machine is 99.99 percent nonterrestrial-origin by mass and requires one new Earth-imported kilogram of control electronics per replacement cycle. Its mass closure is impressive. If no in-boundary controller substitute exists, its capability closure for an indefinitely renewable machine is incomplete.

This is not an argument that every low-mass input is equally important. It is an argument for publishing the vector. Three complementary summaries answer different questions:

1. **Embodied mass closure** C_m reports recursively attributed nonterrestrial material provenance for a declared output.
2. **Critical-input capability closure** $C_{\text{crit}}^{\text{cap}}$ identifies the least stress-supplied quantitative class in a versioned essential set.
3. **Autonomy horizon** H estimates how long the system can meet requirements under an explicit

disruption scenario.

For a class with a net deficit $D_j = [R_j - L_j - V_j - U_j]_+$ and usable stock S_j, a first-order stockout horizon is

$$H_j = \begin{cases} S_j/D_j, & D_j > 0, \\ \infty, & D_j = 0, \end{cases} \qquad H = \min_j H_j.$$

This equation omits stochastic consumption, repair queues, substitution, and transport lead time; operational models should include them. Its purpose is to show why closure and resilience are related but not identical. A system can have low local closure and a long autonomy horizon because it holds large imported inventories. It can have high nominal closure and a short horizon because one rare item has no buffer.

Closure should also be evaluated under at least three targets:

- **operate:** sustain current output and routine maintenance;
- **recover:** restore service after declared failures or loss of a node; and
- **expand:** commission a specified increment of complete-equivalent capacity.

A system closed for operation but not recovery is brittle. A system closed for operation and recovery but not expansion can be space-sustaining without being self-expanding.

4.5 The import-ceiling result

Closure becomes a scaling constraint when an essential import channel is bounded. Consider a long-run window with no net inventory draw, $\dot{S}_j = 0$, and with losses included in the input coefficients. Suppose new imports of quantitative class j cannot exceed the fixed absolute cap $\bar{U}_j$. Define the sustainment margin

$$F_j(K) = L_j(K) + V_j(K) + \bar{U}_j - a_j^M K - d_j.$$

Here $L_j(K)$ and $V_j(K)$ are net qualified flows after internal intermediate demand and recursive upstream attribution; the same boundary-crossing input cannot appear inside either term and again as available U_j. The installed base is feasible under the cap only if $F_j(K) \geq 0$ for **every** essential quantitative class, including classes with $a_j^I = 0$. If any $F_j < 0$, there is no feasible maintenance-and-demand solution at that scale; clipping the result to zero commissioning would conceal an already unsustainable installed base. Conditional on sustainment feasibility, gross commissioning must satisfy

$$0 \leq I_K \leq \min_{j:a_j^I>0} \frac{F_j(K)}{a_j^I}.$$

If the index set $\{j : a_j^I > 0\}$ is empty, this particular commissioning bound is defined as $+\infty$; the sustainment tests still apply and some other resource constraint must bound commissioning. If permanent retirements occur at fractional rate δ, then

$$g_{\max} \leq \frac{1}{K} \min_{j:a_j^I>0} \frac{F_j(K)}{a_j^I} - \delta,$$

again only after every sustainment constraint passes. Finite stored inventory can bridge a violation for a finite time; its depletion must be modeled explicitly and cannot establish a long-run regime.

To write the same condition directly in terms of capability closure, adopt the Earth-versus-all-nonterrestrial boundary so that $V_j = 0$, and use the same stress case, import cap, and jointly feasible supply vector that defines $\mathbf{c}^{\text{cap}}$. For a target g, gross accepted incremental capacity is $I_K = (g+\delta)K$. If the declared fallback is fully dispatched so that $L_j = \widehat{L}_{j,\text{stress}}$, and if there is exact balance, $d_j = 0$, no inventory draw, and $L_j < R_j$, then the required new Earth supply is

$$U_{j,\text{required}} = (1 - c_j^{\text{cap}}) \left[a_j^M + a_j^I(g+\delta)\right] K.$$

A bounded channel therefore imposes

$$K \le \frac{\bar{U}_j}{(1 - c_j^{\text{cap}}) \left[a_j^M + a_j^I(g+\delta)\right]}.$$

If the fallback is available but not dispatched, the right-hand side of the import equation is the minimum new Earth flow achievable by full fallback dispatch, not the actual import flow. This is the paper's central conditional scaling result. It applies at a stated $g > 0$ over a range in which the reference basket and boundary are fixed, the jointly dispatchable nonlocal fraction $1 - c_j^{\text{cap}}$ and the relevant input intensity remain bounded away from zero, new Earth imports are bounded, and Earth-origin inventories are finite with losses accounted. For each affected recurring class, continued unbounded expansion at that fractional rate then requires at least one of the following:

1. c_j^{cap} approaches one fast enough as K grows;
2. the input coefficient falls, or the input is eliminated, substituted, or designed out; or
3. the Earth import channel grows with the industrial system, in which case growth is not Earth-decoupled under the fixed-cap definition.

The result does **not** mean closure is always today's active bottleneck. Early systems may be limited by delivered power, excavation, transport, capital, reliability, or demand. It means closure governs whether growth can decouple from Earth. Throughput and power determine how much the system can do. Closure determines whether terrestrial support must scale with it.

4.6 Closure is not autarky

The framework does not assign moral value to local production. Imports may remain cheaper, safer, or strategically desirable. A high-closure system might intentionally buy advanced electronics from Earth while maintaining an inferior local fallback. A distributed solar economy might trade among the Moon, asteroids, Mars, and orbital habitats; "local" depends on the reporting boundary.

The relevant program claims are nested or orthogonal, not mutually exclusive, and should therefore be reported separately:

- **Bounded-import sustainment:** the system can maintain $g \ge 0$ for a stated horizon under per-class fixed absolute caps $U_j \le \bar{U}_j$.
- **Campaign self-expansion:** the system maintains $g \ge g_{\min} > 0$ across repeated commissioning lags under those caps or under a preregistered absolute cap schedule whose supremum is finite and independent of realized K.
- **Asymptotic Earth-decoupling:** new Earth flow is $O(1)$ for every recurring essential class as K grows. For an Earth-versus-all-nonterrestrial boundary ($V_j = 0$) with fixed input intensities, this is equivalent to each new-Earth requirement $(1 - c_j^{\text{cap}})K$ remaining bounded.
- **Finite-horizon no-Earth-resupply:** the system can operate, recover, and meet the declared target for a stated horizon with $U_j = 0$, possibly by drawing previously imported inventory or receiving V_j from outside the geographic boundary.

- **Renewable boundary autonomy:** the system can continue with $U_j = V_j = 0$ and without net depletion of finite external-origin inventory while all quantitative and categorical acceptance tests remain satisfied.

Only the last is technical autarky. Declining import **intensity** alone is insufficient for Earth-decoupling: an intensity proportional to $K^{-1/2}$ still permits absolute imports to grow as $K^{1/2}$.

5 Bootstrap leverage and growth

5.1 Capacity balance

Let $I_K(t) = I_K^{\text{new}}(t) + I_K^{\text{upgrade}}(t)$ denote accepted incremental installed CE from new commissions and permanent capability upgrades. Let permanent retirement or derating be $R_K(t) = \delta(t)K(t)$. The installed-capital balance is

$$\dot{K}(t) = I_K(t) - \delta(t)K(t).$$

Temporary outage, repair, and reactivation are reconciled in K_{avail} and never counted as I_K. This identity becomes a model only after I_K is connected to materials, power, process capacity, allocation, yield, provenance, and delay. If μ_K is accepted material required per CE and ε_K is commissioning energy per CE, aggregate resource bounds include

$$I_K \leq \frac{r_m \dot{m}_{\text{qualified}}}{\mu_K} \qquad \text{and} \qquad I_K \leq \frac{r_P P_{\text{industrial}}}{\varepsilon_K},$$

where r_m and r_P are the fractions of qualified material and industrial energy allocated to expansion. These are upper bounds, not quantities to add. The actual commissioning rate is set by the binding complementary input.

For input allocations x_j^I,

$$I_K(t+\tau) \leq \min_{j:a_j^I>0} \left(\frac{x_j^I(t)}{a_j^I} \right),$$

where τ is the interval from committing inputs to accepting productive capacity. Because a_j^I is defined per **accepted** CE, it already includes process yield, rejects, and qualification loss; multiplying by another yield would count the loss twice. A model that sends material into a build queue and credits capacity instantly overstates growth.

5.2 A reduced delay model

Suppose all binding supplies and source-qualified expansion allocations scale with installed capacity. Let q be the gross nonterrestrial-origin reproduction coefficient, in inverse time, before delay. Then

$$I_K(t) = qK(t-\tau)$$

and

$$\dot{K}(t) = qK(t-\tau) - \delta K(t).$$

For a constant exponential solution $K(t) \propto e^{gt}$ after an admissible initial history,

$$g + \delta = qe^{-g\tau}.$$

Equivalently,

$$g = \frac{W\left(q\tau e^{\delta\tau}\right)}{\tau} - \delta,$$

where the dominant real characteristic exponent uses the principal branch of the Lambert W function. With zero delay, $g = q - \delta$. When $q > \delta$ so that growth is positive, a positive delay produces a smaller dominant growth rate for the same q and δ because capacity committed today cannot commission descendants until later. For non-growing cases, the sign and dominant history-dependent mode should be evaluated directly rather than inferred from that verbal comparison.

The exponential solution is conditional. It assumes stationary input intensities, yields, allocation fractions, resource access, failure rates, and commissioning delays; adequate demand; and no external constraint becoming tighter with scale. The equation is a local regime model, not a centuries-long forecast.

5.3 Bootstrap leverage $\mathcal{B}$

The landing-page draft used β for bootstrap leverage. This paper uses $\mathcal{B}$ because β is already conventional notation for a solar sail's lightness number, a parameter needed in the architecture analysis.

For campaign $[t_0, t_1]$, define gross bootstrap leverage as

$$\mathcal{B}^{\text{gross}}_{[t_0,t_1]} = \frac{\int_{t_0}^{t_1} I_{K,\text{NT}}(t)\,dt}{K_{\text{seed,Earth}}}.$$

Here $I_{K,\text{NT}}$ is accepted, source-attributed nonterrestrial-origin incremental capacity from new commissions or permanent upgrades; it never includes repair or reactivation of the same asset. Both numerator and denominator are complex-equivalents, so $\mathcal{B}$ is dimensionless. A mixed-origin CE must carry provenance across its entire versioned basket. A report either applies a preregistered minimum expansion-closure threshold to count a whole CE or publishes a closure-weighted CE numerator alongside the full process-source allocation. Making only the final bottleneck component nonterrestrial-origin does not convert legacy Earth-origin capacity into a nonterrestrial CE. The report must also state:

- campaign duration and capacity basket;
- whether the numerator is gross commissioned capacity or surviving capacity;
- which seed assets count in the denominator;
- continuing imported material and components;
- whether descendant capacity is included; and
- the recursive provenance and minimum expansion-closure vector of commissioned capacity.

$\mathcal{B}$ is cumulative leverage, not a growth rate. A campaign can have positive or large gross leverage while every source-qualified commission merely replaces retirement. A system can also report impressive leverage while depending on a small stream of imported critical components. Therefore $\mathcal{B}$ must be read with g and $\mathbf{C}$.

5.4 Relation between leverage and growth

Under the reduced zero-delay model with constant q, constant δ, no later Earth capacity additions, and $K(0) = K_0$,

$$K(T) = K_0 e^{gT}, \qquad g = q - \delta.$$

Cumulative gross source-qualified commissioned capacity is

$$\int_0^T qK(t)\,dt = \frac{qK_0}{g}\left(e^{gT} - 1\right)$$

for $g \neq 0$. Thus

$$\mathcal{B}_T^{\text{gross}} = \frac{qK_0}{gK_{\text{seed,Earth}}}\left(e^{gT} - 1\right),$$

with $\mathcal{B}_T^{\text{gross}} = qTK_0/K_{\text{seed,Earth}}$ when $g = 0$. Net installed gain relative to the Earth seed is

$$\mathcal{B}_T^{\text{net}} = \frac{K(T) - K_0}{K_{\text{seed,Earth}}} = \frac{K_0}{K_{\text{seed,Earth}}}\left(e^{gT} - 1\right).$$

The simpler expressions $\frac{q}{g}(e^{gT} - 1)$ and $e^{gT} - 1$ require the additional assumption $K_0 = K_{\text{seed,Earth}}$. If $\delta = 0$, gross and net leverage coincide under consistent source attribution. With retirement, gross leverage exceeds net leverage because some commissioning replaces lost capacity. These equations show precisely why $\mathcal{B}$ does not determine g without the seed relation, campaign duration, retirement rate, timing convention, and gross-versus-net definition.

A discrete commissioning-cycle model provides the same lesson. If fraction s of parent capacity survives one cycle of duration τ and each parent commissions R CE per parent CE during that cycle, then R is dimensionless and

$$K_{n+1} = (s + R)K_n, \qquad g = \frac{\ln(s + R)}{\tau}.$$

Expansion requires $s + R > 1$. It does not require a parent to build a full copy every cycle, because the parent usually survives and partial capacity accumulates. Conversely, $R = 1$ does not imply rapid growth if the cycle is long or survival is poor.

5.5 Illustrative parameter sets

The following cases are diagnostic examples, not predictions. All capacity is measured against one unchanged reference basket; leverage examples assume $K_0 = K_{\text{seed,Earth}}$ and that gross commissioning meets the declared source-qualification rule.

Case	Key assumptions	Result	Interpretation
Bulk-rich, vitamin-poor	$K = 100$ CE; $C_m = 99.9\%$; controller imports cap I_K at 1 CE/yr; $\delta = 2\%$/yr	$g(t_0) = -1\%$/yr	Instantaneous result at the stated scale; high mass closure conceals contraction

Case	Key assumptions	Result	Interpretation
Sustaining, not expanding	$q = 2\%/\text{yr}$; $\delta = 2\%/\text{yr}$; $T = 10$ yr	$g = 0$; $\mathcal{B}^{gross} = 0.20$	Local commissioning only replaces retirement
Expanding, zero delay	$q = 15\%/\text{yr}$; $\delta = 3\%/\text{yr}$; $T = 10$ yr	$g = 12\%/\text{yr}$; $t_2 = 5.8$ yr; $\mathcal{B}^{gross} = 2.90$	Idealized proportional growth
Same plant, two-year delay	$q = 15\%/\text{yr}$; $\delta = 3\%/\text{yr}$; $\tau = 2$ yr	$g \approx 9.4\%/\text{yr}$; $t_2 \approx 7.4$ yr	Delay alone removes about one-fifth of asymptotic growth

The first case is the closure argument in miniature. The second separates reproduction from expansion. The third establishes a transparent ideal baseline. The fourth demonstrates why commissioning lead time belongs in any serious growth claim.

5.6 What can break exponential growth

Even if $g > 0$ in one operating range, indefinite exponential extrapolation is usually unjustified. Growth can slow because:

- ore grade or accessible resource geometry changes;
- transport distance and inventory in transit increase;
- high-quality sites, orbits, and thermal sinks become scarce;
- collision avoidance and beam coordination consume more capacity;
- utilization falls because demand does not scale with supply;
- a high-yield process encounters a feedstock or quality limit;
- reliability changes with fleet age or design complexity;
- governance restricts replication, resource use, or hazardous power transfer; or
- technology changes invalidate the reference basket.

Uncertain and correlated failures add another caution. A mean g can conceal a substantial chance of contraction or extinction, so a program should report outcome distributions and downside cases rather than presenting an expected rate alone. The reduced deterministic model is most useful for identifying which assumption a scenario needs and which measurement could disprove it.

6 A dependency model, not a calendar

6.1 Why capabilities advance together

The system cannot be decomposed into a neat sequence in which mining finishes, then power begins, then transportation begins, and autonomy arrives last. Every industrial node requires some material supply, energy, logistics, control, and maintenance from its first day. The balance changes with scale, but the tracks mature concurrently.

The dependency model uses five tracks:

1. **Materials:** extraction, beneficiation, refining, feedstock production, fabrication, assembly, qualification, and recycling.
2. **Energy:** generation, storage, transmission, conversion, load management, and heat rejection.

3. **Transport:** launch, surface movement, orbital transfer, propellant or momentum exchange, docking, and freight scheduling.
4. **Autonomy:** sensing, planning, fault detection, repair, reconfiguration, commissioning, and secure distributed control.
5. **Human systems:** supervision, habitation where chosen, medicine, governance, law, finance, safety, and institutional legitimacy.

The fifth track is not a physical prerequisite for an uncrewed machine network. It is included because any real program operating hazardous, valuable infrastructure at planetary scale exists inside human institutions, and because some architectures explicitly include permanent populations. Reports should distinguish a robotic closure requirement from a chosen settlement requirement.

6.2 Regimes as conjunctive tests

Regime evidence combines unlike quantitative thresholds and categorical gates, so it should not be compressed into a ratio that implies false precision. For criterion i and regime r, define a preregistered acceptance predicate

$$A_{i,r}(\text{window}, \text{stress}, \text{basket}) \in \{0, 1\}.$$

The regime is accepted only when

$$A_r = \bigwedge_i A_{i,r} = 1$$

over the declared dwell period, stress case, and versioned basket. Excess power does not compensate for a missing controller; a superb refinery does not compensate for unavailable freight; a high-autonomy demonstration does not compensate for negative installed-capacity growth.

Regimes are conditional and reversible, not permanent badges. For one declared basket, window, and stress case, report the **highest satisfied level** in the ordered ladder below: Earth-supported is the default when bounded-import sustainment fails; space-sustaining adds sustainment and recovery; self-expanding adds repeated positive net expansion; coordination-limited adds evidence that another system constraint has displaced closure as the leading limit. The same network may therefore be space-sustaining for its maintenance basket but Earth-supported for an expansion basket, and it may lose a level after a technology change, depleted inventory, or failed stress test.

6.3 Four operating regimes

6.3.1 Earth-supported

The system does not pass bounded-import sustainment and recovery for the declared current-service basket, horizon, and stress case. In-space extraction and production trials can be substantial, and deployment can be rapid, but once allowed imports and declared starting inventory are exhausted, one or more necessary acceptance predicates fail with no in-boundary recovery path.

In this regime, terrestrial launch is part of the production function. The most useful progress is not a claim of "self-sufficiency" but a measured reduction in imported input intensity per unit of accepted output.

6.3.2 Space-sustaining

The system passes bounded-import sustainment and recovery for the declared service basket and horizon, but has not passed campaign self-expansion for the corresponding expansion basket. It has in-boundary production of bulk materials and routine spares, credible repair, and enough power and transport to keep the installed base productive. Net growth may be zero. Expansion may still require Earth-supplied machine tools, semiconductor devices, specialized labor, or other critical inputs.

"Sustaining" is always conditional on the horizon and stress case. A one-year demonstration with stored spares is not equivalent to a decade with rolling inventory replacement.

6.3.3 Self-expanding

The system demonstrates $g \geq g_{\min} > 0$ across multiple commissioning delays while every new Earth input remains within a preregistered fixed absolute per-class cap $U_j \leq \bar{U}_j$, or a time-varying absolute cap whose supremum is finite, declared in advance, and independent of realized K. Source-qualified commissioned capacity meets the same reference-basket acceptance tests as the installed base. Maintenance, permanent retirement, commissioning, and process qualification are included. A rare import can remain, but observed expansion cannot require it to exceed the bounded cap.

The strongest evidence is not one child factory. It is several overlapping commissioning cohorts, because descendant timing, retirement, and hidden terrestrial support become visible only over repeated cycles.

6.3.4 Coordination-limited

The system continues to satisfy the self-expanding predicates, while measured sensitivity or constrained optimization shows that relaxing closure or Earth-import caps no longer produces the largest feasible increase in g. The leading constraints have shifted toward orbital occupancy, transport distance, thermal rejection, beam and spectrum coordination, security, resource allocation, environmental externalities, or governance. This regime does not mean "finished" and does not require capturing all sunlight. It means growth is primarily limited by coordinating a large solar-system population rather than by reproducing its basic industrial functions. The landing page's shorter label, "swarm-scale," refers to this coordination-limited condition rather than to a collector count.

6.4 The dependency matrix

The matrix below is a compact hypothesis about co-maturation, not a roadmap date or technology-readiness chart. Each cell describes the characteristic service level of a regime. Actual acceptance criteria must be quantitative and architecture-specific.

Table 1: Dependency tracks across operating regimes. Acceptance is basket-, window-, and stress-specific.

Track	Earth-supported	Space-sustaining	Self-expanding	Coordination-limited
Materials	Imported machinery; in-space extraction, separation, and process trials.	Qualified bulk materials and routine spares produced in-boundary; critical imports remain.	Source-attributed complete-equivalent capacity is commissioned under fixed per-class Earth-import caps.	Multi-site supply chains feed continuous construction, recycling, and technology turnover.
Energy	Local arrays serve early loads; major generation and storage hardware is imported.	Distributed generation, storage, and load control sustain operations; beam links appear where link budgets justify them.	Generation, storage, transmission, and thermal systems reproduce with the industrial base.	Multi-orbit energy networks allocate power where aperture, range, pointing, duty-cycle, efficiency, and heat constraints close.
Transport	Terrestrial launch and imported vehicles establish routes.	Reusable in-boundary freight, depots, and nonterrestrial propellant or momentum services reduce resupply.	Source-attributed fleets commissioned under the cap move expansion mass; route capacity grows with industry.	Autonomous logistics, traffic allocation, salvage, and debris control operate across the inner system.
Autonomy	Teleoperation and bounded automation; Earth resolves major faults.	Local fault detection, routine repair, and remote reconfiguration support a stated autonomy horizon.	Commissioning, recovery, and maintenance close across repeated lags with limited supervision.	Distributed control, verification, cyber resilience, and graceful degradation become utilities.
Human systems	Short-duration or remote supervision; Earth institutions retain direct operational control.	Persistent institutions and, where chosen, crews support sustained operations and safety.	Governance scales with industry, resource rights, hazardous beams, and growing settlements where present.	Legitimate multi-party coordination manages infrastructure whose failures and externalities cross communities.

6.5 Assumptions embedded in the matrix

The matrix rests on assumptions that should travel with it.

Qualified output, not demonstrations. A capability counts when it produces accepted service at the relevant duty cycle and environment. One part or one batch is evidence, not sustained capacity.

Bottleneck transitions. The least mature critical track controls the regime. Strong performance elsewhere can create margin but cannot erase a missing necessary service.

Finite reliability. Equipment fails, degrades, drifts, and becomes obsolete. Maintenance and retirement are inside the boundary.

Explicit logistics. Material in a mine, power at a collector, and a spare at another orbit are not available until transport and scheduling deliver them.

Bounded Earth support. Campaign self-expansion is evaluated under declared fixed absolute per-class import ceilings or a preregistered time-varying absolute cap whose finite supremum is independent of realized K, not merely declining intensity or a period of rapid terrestrial resupply.

No magical autonomy. Remote labor, model updates, design services, and authority are reported as dependencies. Autonomy is a portfolio of bounded functions, not a binary label.

No mandatory settlement sequence. Human habitation can grow with industry, lag it, or remain absent at some sites. The matrix treats institutional capability as necessary and permanent crews as architecture-dependent.

A network, not a synchronous grid. "Interplanetary grid" means an asynchronous combination of local generation, storage, directed energy, data, and physical energy carriers. Light-time, pointing, and orbital motion prevent a simple extension of a terrestrial alternating-current grid.

Technology baskets can change. Regime evidence must be rebased when a new architecture changes the meaning of one complex-equivalent. Improvement is welcome; silent denominator changes are not.

7 Architecture consequences

7.1 Swarm as the industrial backbone

An orbital swarm uses the ordinary solution to gravity: free fall. In an ideal two-body model with negligible radiation pressure, a collector inserted into a Keplerian orbit requires no continuous force to avoid falling into the Sun. For a solar sail with constant $\beta_{\text{sail}} < 1$ and radially directed radiation pressure, a corresponding bound orbit exists under the reduced effective solar gravity. This passive orbital support across a wide range of areal densities is the swarm's primary scaling advantage. It permits heterogeneous units, and one unit's mechanical failure does not transfer structural load across a shell.

"Passive" does not mean maintenance-free or automatically safe. Radiation pressure, planetary encounters, nonuniform mass loss, mutual perturbations, attitude errors, and manufacturing dispersion alter trajectories. Dense families require conjunction prediction, orbit allocation, collision tolerance, and end-of-life disposal. Mechanical load transfer is local, but trajectory loss is not: a failed unit remains on a path that can threaten other assets.

The swarm architecture also matches industrial growth. A factory can make the next collector without solving the final population geometry. New unit designs can coexist with old ones. Capacity can be added where flux, temperature, resources, transport, and demand make sense. The population can remain optically thin indefinitely or become denser in selected bands.

7.2 Radiation-supported elements

A statite uses radiation pressure to offset gravity rather than relying on orbital angular momentum. Forward formalized the statite as a spacecraft held at a nonorbiting equilibrium by light pressure, while standard solar-sail dynamics provide the lightness-number and attitude framework used here (Forward 1991; McInnes 1999). For a thin surface normal to sunlight, the incident solar flux is $L_\odot/(4\pi r^2)$. If $Q_{\rm pr}$ is the radiation-pressure efficiency and σ_A is total areal mass density, radiation acceleration is

$$a_{\rm rad} = \frac{L_\odot Q_{\rm pr}}{4\pi r^2 c \sigma_A}.$$

Solar gravity is

$$a_g = \frac{GM_\odot}{r^2}.$$

Define the solar-sail lightness number

$$\beta_{\rm sail} = \frac{a_{\rm rad}}{a_g} = \frac{L_\odot Q_{\rm pr}}{4\pi c G M_\odot \sigma_A}.$$

Both accelerations scale as r^{-2} in the ideal point-source model. Radial balance requires $\beta_{\rm sail} = 1$, giving a critical total areal density

$$\sigma_{A,\rm crit} = \frac{L_\odot Q_{\rm pr}}{4\pi c G M_\odot} \approx 0.765\, Q_{\rm pr}\ \mathrm{g\,m^{-2}},$$

using the IAU nominal solar luminosity and solar mass parameter (Prša et al. 2016). $Q_{\rm pr}$ is near one for an ideal absorber and can approach two for ideal specular reflection at normal incidence. The density includes film, conductors, structure, controls, and payload. This is an exceptionally demanding mass budget.

Attitude changes provide force-direction control, but they also change projected area, optical loading, and equilibrium. Finite solar angular size, nonideal reflectivity, degradation, thermal distortion, shadows, and interactions among units complicate the point model. A radiation-supported precision layer is therefore not a generic replacement for orbiting industrial mass. It is a specialized architecture for ultralight elements whose geometric position is worth continuous control complexity.

7.3 Stability is conditional

A sparse collection of isolated statites should ordinarily be treated as an actively controlled system. In the ideal point-source radial model, exact $\beta_{\rm sail} = 1$ balance is neutral rather than asymptotically stable: displacement does not create a restoring radial force and a velocity perturbation can produce drift. Finite-star and extended-disc static equilibria analyzed by McInnes are linearly unstable, while lateral behavior of a controlled sail depends on attitude and control law (McInnes 1999; McInnes 2026). Any change in areal density or optical response changes the balance. A unit that loses control has no universal failure trajectory; its motion depends on residual radiation pressure, attitude, velocity, and local perturbations.

Recent analysis adds an important qualification. McInnes finds that idealized dense clouds of radiation-supported elements can become passively self-stabilizing through collective optical and gravitational effects under restrictive model assumptions, even though the point-source isolated equilibrium is not asymptotically stable and the finite-star or extended-disc isolated equilibria analyzed there are unstable (McInnes 2026). That result makes "a bubble can never be passively stable" too broad. It does not make a practical dense bubble proven. Optical depth, collisions, finite elements, control authority, deployment, and structural areal density remain engineering questions.

The correct architectural statement is therefore conditional:

- conventional sparse statite populations require active control;
- collective passive stability may exist in special dense configurations; and
- orbital units retain a far more mature passive-support mechanism for bulk infrastructure.

7.4 Hybrid division of labor

The proposed baseline is **orbit for scale, radiation support for selected geometry**.

Orbital units carry mines, factories, storage, most generation, computation, habitats, transport infrastructure, and large radiators. Radiation-supported or actively displaced units may serve as mirrors, sunshades, relays, observatory elements, navigation references, or transmitters whose position matters more than areal mass. Active low-thrust displaced orbits add another option between pure Keplerian motion and pure radiation balance.

This is not a permanent prescription. It is the consequence of current physical scaling. For $\beta_{\text{sail}} < 1$, orbital support accommodates a much wider range of areal densities through angular momentum. At $\beta_{\text{sail}} \geq 1$, a fully illuminated normal sail must reduce effective radiation pressure through attitude, optical control, or active thrust to remain bound. Pure statite balance supports only a narrow relationship among optical properties and areal density. A hybrid architecture lets each mechanism do the job for which its constraint is least damaging.

7.5 Thermal accounting

Collection is only one side of the energy balance. For a declared control volume, conservation requires

$$P_{\text{absorbed}} + P_{\text{imported}} = P_{\text{exported}} + P_{\text{rejected}} + \frac{dE_{\text{boundary}}}{dt}.$$

Here imported and exported power include directed beams and the energy or enthalpy carried by matter crossing the boundary; P_{rejected} excludes energy already classified as an export. E_{boundary} includes electrical, thermal, chemical, kinetic, and other stored energy. Useful work performed and dissipated inside the boundary ultimately appears as heat. A radiator with planform area A_R, n effective radiating faces, emissivity ϵ, and temperature T_R emits

$$P_{\text{emit}} = n\epsilon\sigma_{\text{SB}} A_R T_R^4.$$

This is emitted power, not necessarily net rejected power. Net rejection subtracts absorbed sunlight, mutual irradiation, and background loading through the actual view factors. Only for a uniform effective sink can it be approximated by $n\epsilon\sigma_{\text{SB}} A_R (T_R^4 - T_{\text{sink}}^4)$. Degradation and temperature limits further reduce usable performance.

Moving inward raises incident solar flux as r^{-2} and can reduce collector area for a given incident power. It also raises equilibrium temperature, radiation environment, and thermal-control difficulty. A program that reports collected watts without radiator area, temperature, and sink geometry has omitted a first-order constraint. At large scale, thermal design is part of architecture selection, not a downstream component choice.

8 System constraints beyond the dashboard

8.1 Autonomy and maintenance

Communication delay does not by itself require general machine intelligence. It does require local handling of faults that evolve faster than a human round trip, and it makes remote micromanagement increasingly costly with distance and fleet size. The relevant autonomy portfolio includes state estimation, anomaly isolation, safe shutdown, inspection, repair planning, manipulation, process control, inventory management, commissioning, and authenticated software change.

Autonomy claims should be reported by bounded task and recovery envelope. "Human in the loop" can describe anything from occasional approval to continuous joystick control. Functional closure should count remote labor intensity, specialist response time, and Earth-only tools. If supervision hours rise linearly with K, human operations become a scaling dependency even when physical imports are small.

Maintenance is productive output, but not new installed capital. A repaired tug, recalibrated sensor, or recovered furnace can restore K_{avail} without changing K_{inst} and without increasing $\mathcal{B}$. Reports should separate preventive maintenance, corrective repair, temporary outage and reactivation, cannibalization, permanent retirement, and genuinely new commissioning so that repeated repair of one asset cannot manufacture apparent reproduction.

8.2 Orbital capacity and debris

A solar-orbiting swarm does not reproduce the exact debris environment of low Earth orbit. Heliocentric volumes, relative velocities, resonances, perturbations, and removal mechanisms differ. The relevant transferable lesson is qualitative: collision risk can be endogenous because more objects create more conjunctions and fragmentation can create more hazardous objects. Kessler and Cour-Palais modeled that feedback for artificial satellites in Earth orbit (Kessler and Cour-Palais 1978). Dedicated work on dense heliocentric collector populations likewise finds that collisional cascades can constrain proposed megaswarms, with results dependent on unit size, optical depth, velocity distribution, and removal physics (Lacki 2025). A solar-scale program therefore needs architecture-specific collision kernels and occupancy models rather than transplanted low-Earth-orbit thresholds.

At minimum, orbit governance must specify ephemeris quality, maneuver authority, keep-out volumes, loss-of-control behavior, passivation, salvage rights, and debris responsibility. The safe carrying capacity of an orbital family depends on unit area-to-mass ratio, relative velocity, tracking uncertainty, maneuverability, and fragmentation behavior, not merely collector count.

8.3 Energy networks

Generation and demand can be separated by millions of kilometers and by moving geometry. A useful network may combine local electrical systems, storage, microwave or optical beams, transported fuels, momentum exchange, and demand scheduling. Interplanetary beaming is an architecture hypothesis, not a default grid primitive. Established microwave link accounting makes frequency, transmitter and receiver apertures, range and geometry, pointing, and end-to-end efficiency explicit (Brown and Eves 1992). This framework additionally requires duty cycle, exclusion constraints, and waste heat; optical links require their own wavelength-specific component and propagation model. Delivered power P_L remains indexed by destination, time, and quality.

Directed energy also couples engineering to governance. A beam capable of moving industrial power is a hazardous actuator. Pointing assurance, sidelobes, authentication, independent shutdown, exclusion geometry, audit logs, and liability are system requirements. A grid that cannot establish who may energize which receiver is not operationally closed.

8.4 Security and fault containment

Self-expansion magnifies both productive and destructive mistakes. The objective is not one perfectly trusted controller. It is containment: bounded authority, verified commands, diverse implementations, rate limits, physical interlocks, compartmentalized production, recoverable state, and observability across organizations.

Security closure belongs in **C** when the system cannot safely operate without Earth-based keys, model services, or incident response. At the same time, full local control without legitimate oversight may reduce technical dependence while increasing political risk. The model reports the dependency; governance decides whether it is acceptable.

8.5 Governance and human systems

Existing space law already assigns responsibilities to states for national space activities and requires authorization and continuing supervision; the Outer Space Treaty also establishes due-regard and harmful-interference obligations (United Nations 1967). Current United Nations long-term-sustainability guidelines are voluntary and non-legally binding; they address registration, orbital information, conjunction assessment, debris, regulatory frameworks, and information sharing (United Nations Committee on the Peaceful Uses of Outer Space 2019). Neither instrument was written for a Dyson swarm, but both show that coordination is not optional even at present scale.

Future institutions would need to govern resource access, beam corridors, orbital occupancy, environmental reserves, salvage, liability, replication controls, cyber incidents, scientific quiet zones, and the standing of remote or permanent communities. Those institutions need not be centralized. They do need legitimate authority, interoperable records, dispute resolution, and enough technical visibility to enforce shared constraints.

The framework takes no position on who should own stellar-scale infrastructure. It makes a narrower systems claim: governance is a functional dependency because unmanaged externalities can bind growth as decisively as a missing material.

9 Measurement, demonstration, and falsification

9.1 A minimum reporting package

A claim of industrial progress should be reproducible from a compact evidence package. At minimum, publish:

1. **Boundary and window:** locations, dates, starting inventories, external services, and stress case.
2. **Material balance:** stage-specific new Earth, external non-Earth, recycled Earth-origin, virgin nonterrestrial, recycled nonterrestrial-origin, rejected, inventoried, and accepted flows, with geography and provenance reported as separate ledgers.
3. **Power balance:** allocated generation, storage state, imports and exports, conversion and transmission losses, metered delivered power, firm power, boundary energy change, and net thermal rejection.

4. **Closure bill:** realized and capability vectors, categorical predicates, recursive attribution, inventories, preregistered substitutions, non-substitutable cut sets, and versioned criticality set $\mathcal{E}$.
5. **Capacity record:** installed and available process vectors, reference basket and allocation model, resulting K_{inst} and K_{avail}, binding constraint, and basket version.
6. **Growth reconciliation:** new accepted commissions, permanent retirements and derates, temporary outages and reactivations, source attribution, commissioning delays, $\bar{g}_{\log}$, CAGR, and g_{flow}.
7. **Bootstrap reconciliation:** campaign definition, Earth seed, source-qualified gross and net commissioning, continuing imports, full-CE provenance convention, and $\mathcal{B}$ convention.
8. **Reliability record:** exposure hours, failure modes, repair times, cannibalization, losses, and uncertainty.
9. **Information and labor dependencies:** remote operator hours, specialist interventions, Earth-only software or design services, and authority dependencies.
10. **Externalities:** collision risk, beam safety, waste streams, environmental effects, and incidents.

The package is intentionally harder to satisfy than a publicity metric. Its purpose is to let another team reconstruct why a claimed regime follows from measured behavior.

9.2 Demonstrations that change the model

Useful demonstrations form a ladder of increasing system closure. The rungs are not dates and do not require a particular destination.

9.2.1 Qualified in-space output

Produce a specified material or part from declared feedstock in the relevant environment and measure source provenance, energy, yield, rejects, equipment wear, and imported consumables. The key transition is from "process occurred" to a repeatable accepted-output rate.

9.2.2 Nonterrestrial maintenance input

Use source-attributed nonterrestrial output to restore a productive asset to service. This tests whether qualification standards reflect real function and can raise K_{avail}; it does not count as new installed capacity or bootstrap leverage.

9.2.3 Bounded-import sustainment

Operate an integrated production network for a stated horizon under an enforced import ceiling. Replace routine wear items, manage inventory, and recover from a declared set of faults. This is stronger evidence than nominal mass closure because it exercises timing and complementarity.

9.2.4 Complete-equivalent commissioning

Commission new capacity against the same reference basket used for the parent system. Recursively attribute the entire basket and measure the delay from committed resources to accepted service. One in-space machine, or one nonterrestrial bottleneck component attached to Earth-origin capital, is useful capacity but not a wholly nonterrestrial CE.

9.2.5 Positive net growth across repeated lags

Maintain $g \geq g_{\min} > 0$ over multiple commissioning cohorts after permanent retirement and derating, while separately reporting temporary availability. Enforce fixed absolute per-class Earth-import caps or a preregistered time-varying schedule whose finite supremum is independent of realized K. This is the empirical threshold for the paper's campaign definition of self-expansion.

9.2.6 Constraint migration

Demonstrate by sensitivity analysis or constrained optimization that relaxing closure or an import cap no longer gives the largest feasible increase in g, and identify the new leading limit: resource access, orbital occupancy, heat rejection, demand, security, or governance. A real program model should predict not only progress but the class of bottleneck that appears next.

9.3 Falsifiable propositions

The framework makes propositions that can fail.

P1: A scalar mass-closure metric is insufficient. If mass closure alone predicts bounded-import sustainment and expansion across diverse systems as well as a critical-input vector, the vector adds unnecessary complexity. If high-C_m systems repeatedly fail through low-mass dependencies, the proposition is supported.

P2: Critical nonlocal input coefficients do not vanish automatically with scale. For a preregistered basket, boundary, capacity interval, and tolerance ϵ_j, estimate each new-Earth coefficient $U_{j,\text{required}}(K)/K$ while recording substitution and inventory. The empirical proposition is supported when at least one recurring essential coefficient remains above ϵ_j across that interval. Under a fixed cap, that class becomes constraining once $U_{j,\text{required}}(K) > \bar{U}_j$ unless the technology or supply network changes. The proposition is weakened when all preregistered coefficients fall below tolerance through measured yield, substitution, or redesign rather than boundary changes or inventory draw. The ceiling at a fixed positive fractional-growth target is an algebraic consequence of the measured coefficient and cap; whether the coefficient persists is the falsifiable engineering claim.

P3: Commissioning delay materially changes positive realized growth. For campaigns with $q > \delta$, compare cohort timing with a zero-delay model while holding source-qualified reproduction and retirement coefficients fixed. If descendant timing has negligible effect, the delay representation can be simplified. If observed growth falls with longer τ, the model captures a real constraint.

P4: Regime transitions are conjunctive. If a system repeatedly meets a regime target while one preregistered necessary predicate remains false, the acceptance model is too conservative or the predicate was not actually necessary. If failed regime claims cluster at false necessary predicates, the conjunctive model is useful.

P5: The four diagnostic families improve program diagnosis. Compare projects with similar calendars, launch mass, or nameplate power. If $\dot{m}$, P, $\mathbf{C}$, and g do not better explain sustained productive behavior, the dashboard should be revised.

These tests prevent the framework from becoming a vocabulary that explains every outcome after the fact. A failed prediction should change the input classes, capacity basket, constraint form, or the dashboard itself.

9.4 Common reporting traps

Several practices can manufacture apparent progress without changing the system.

- **Stage summation:** adding extracted, refined, and fabricated mass even though they are the same material at different steps.
- **Inventory laundering:** importing supplies before the reporting window and calling subsequent consumption local independence.
- **Assembly laundering:** labeling an in-space assembled product nonterrestrial while ignoring imported upstream components.
- **Recycling inflation:** counting recovered feed and the resulting product as two new material outputs.
- **Installed/available conflation:** reporting installed equipment as current service while omitting power, inputs, transport, derating, or outage.
- **Repair-as-growth:** counting reactivation of the same installed asset as a new commission or bootstrap descendant.
- **Gross-growth substitution:** reporting commissioned capacity while omitting permanent retirement and derating.
- **Denominator drift:** changing the reference basket so that one CE becomes easier to achieve.
- **Descendant double-counting:** crediting the same source-qualified commissioned capacity in multiple leverage generations.
- **Remote-labor omission:** excluding Earth-based operations, design, and troubleshooting from functional dependence.
- **Risk averaging:** reporting expected growth without downside, correlated failure, or loss-of-control scenarios.

The remedy is not a larger headline number. It is a reconciled balance sheet of mass, energy, dependency, and capacity.

10 Research agenda

10.1 Measurement science for productive capacity

The most important unresolved accounting problem is K. A universal industrial capacity unit does not exist. A reference basket must be specific enough to detect missing functions and stable enough to compare campaigns. Research is needed on:

- process-level natural units and acceptance tests;
- substitution-aware aggregation through linear or mixed-integer optimization;
- quality, duty cycle, and availability adjustments;
- rebasing methods when technology changes;
- uncertainty propagation from $\mathbf{k}^{\text{inst}}$ and $\mathbf{k}^{\text{avail}}$ to both CE projections and g; and
- public benchmark baskets for different industrial missions.

One useful approach is to publish several baskets: sustainment, expansion, transport, and energy infrastructure. A system may have $K = 10$ CE against one basket and $K = 2$ against another. That is information, not inconsistency.

10.2 Closure maps and critical vitamins

Closure research should move from total mass fractions to auditable dependency maps. Priorities include:

- recursive bills of material and process chemicals;
- replacement-frequency distributions rather than nameplate composition;

- catalysts, lubricants, seals, bearings, sensors, optics, semiconductor devices, and metrology;
- tooling needed to make the tooling;
- inventory and substitution under long transport lead times;
- embodied software, specialist labor, data, and authorization; and
- graceful degradation when a critical class is unavailable.

The objective is not to make every item locally at any cost. It is to know which import reduction changes the scale ceiling most.

10.3 Commissioning, reliability, and cohort data

Growth models need empirical distributions for yield, delay, and retirement. An industrial demonstrator should record every commissioning cohort from committed inputs through accepted capacity, subsequent uptime, repair, and retirement. Those data permit survival analysis, learning curves, queue models, and stochastic growth estimates.

This research also tests modularity. A system may grow faster by commissioning partial capacity continuously than by waiting for complete cloned factories. The reference basket can reconcile those increments once enough complementary services exist to form another CE.

10.4 Architecture and environmental capacity

The orbital and bubble analysis leaves major open questions:

- safe occupancy of heliocentric orbit families under tracking and maneuver uncertainty;
- collision and fragmentation kernels for high area-to-mass collectors;
- collective optical and dynamical stability of dense radiation-supported clouds;
- degradation and control of sub-gram-per-square-meter functional films;
- multi-temperature radiator placement and mutual view factors;
- beam-network capacity under pointing, diffraction, and exclusion constraints;
- resource extraction limits and environmental preservation zones; and
- salvage and end-of-life pathways that remain effective as the population grows.

These are not details to solve after industrial closure. They determine the technology basket and may become the next binding constraints.

10.5 Governance as a control system

Technical research should treat institutions as part of the operating architecture. Questions include:

- which actions require shared authorization;
- how orbit, beam, and resource claims are recorded;
- how parties verify capacity and closure without exposing every proprietary detail;
- how automatic safety interlocks interact with legal authority;
- how incidents are attributed across long delays and distributed ownership;
- which replication or beam capabilities require rate limits; and
- how communities affected by externalities obtain standing in decisions.

The analogy to a control system is limited but useful: legitimate governance needs observability, authority, feedback, and recovery. A technically closed factory inside an institutionally ungovernable network is not a durable program.

11 Conclusion

A Dyson swarm is easy to draw because the drawing omits the factory.

The factory, in turn, cannot be understood as one replicating robot. It is a network of extraction, refining, fabrication, assembly, transport, power, controls, maintenance, and institutions. The network demonstrates campaign self-expansion only when source-qualified new installed capacity exceeds permanent retirement across repeated lags while every new Earth input stays within a declared absolute cap.

Four headline diagnostic families keep that argument measurable:

- $\dot{m}$ reports accepted physical throughput rather than moved dirt;
- P reports power delivered to work rather than nameplate generation;
- $\mathbf{C}$ reports realized and stress-tested quantitative supply plus categorical gates rather than one reassuring mass percentage; and
- g reports net change in installed productive capacity rather than gross construction or repaired availability.

The installed stock K makes growth measurable, while K_{avail} prevents outage recovery from masquerading as reproduction. Bootstrap leverage $\mathcal{B}$ measures cumulative source-qualified nonterrestrial commissioning per Earth-supplied seed. The import-ceiling result is deliberately conditional: at a stated fixed positive fractional-growth target, if a recurring new-Earth-dependent requirement per unit capacity remains bounded away from zero, new Earth supply is capped, and finite Earth-origin inventory is not being depleted, industrial scale is finite unless that dependency closes, its coefficient falls, or the Earth channel grows with the system.

This does not make industrial closure the first problem to solve. Early programs will be limited by launch, power, process yield, transport, reliability, finance, and demand. It makes closure the condition that separates a large Earth-supported project from an industry capable of producing its own next increment.

The swarm is therefore not the plan. It is what the plan can eventually produce. There is no finished state - only a measured capacity for the next increment.

12 Appendix A: Notation

Symbol	Definition	Units or range
$\dot{m}_s$	Accepted material throughput at stage s	mass/time
P_L	Usable power delivered at load L	W
c_j^{real}	Actual in-boundary supply divided by nonzero requirement	$[0, 1]$
c_j^{cap}	Stress-demonstrated available supply divided by nonzero requirement	$[0, 1]$
A_h	Acceptance predicate for categorical dependency h	$\{0, 1\}$
$\mathbf{C}$	Structured closure record $[\mathbf{c}^{\text{real}}, \mathbf{c}^{\text{cap}}, \mathbf{A}]$	mixed
C_m	Recursively attributed nonterrestrial-origin mass fraction	$[0, 1]$
$C_{\text{crit}}^{\text{cap}}$	Minimum capability closure over versioned essential classes	$[0, 1]$
k_i^{inst}	Accepted installed capacity of process i	process-specific output/time
k_i^{avail}	Currently available capacity of process i	process-specific output/time
b_i	Process-i requirement for one reference basket	same units as k_i/CE
K_{inst}	Installed complete-equivalent stock; written K in derivations	CE
K_{avail}	Available complete-equivalent service	CE
I_K^{new}	Accepted installed capacity from new commissions	CE/time
I_K^{upgrade}	Accepted incremental capacity from permanent upgrades	CE/time
I_K	Sum $I_K^{\text{new}} + I_K^{\text{upgrade}}$	CE/time
$I_{K,\text{NT}}$	Source-qualified nonterrestrial commissioning rate	CE/time
R_K	Permanent capacity retirement or derating rate	CE/time
δ	Fractional retirement hazard, R_K/K	1/time

Symbol	Definition	Units or range
g	Net continuous capacity growth, $d \ln K/dt$	1/time
$\bar{g}_{\log}$	Time-window log growth rate	1/time
q	Gross proportional reproduction coefficient	1/time
τ	Commissioning delay	time
$\mathcal{B}$	Campaign bootstrap leverage	dimensionless
R_j	Required flow of input class j	input-j/time
L_j	Qualified supply produced inside the boundary	input-j/time
V_j	Qualified inflow from non-Earth sites outside the geographic boundary	input-j/time
U_j	New supply crossing the Earth boundary	input-j/time
$U_{j,\text{required}}$	Minimum new Earth flow required by the declared balance	input-j/time
$\bar{U}_j$	Fixed absolute Earth-import cap	input-j/time
S_j	Usable inventory of class j	input-j
a_j^M	Maintenance input intensity	input-j/(CE time)
a_j^I	Commissioning input intensity	input-j/CE
d_j	Other final demand	input-j/time
F_j	Sustainment margin under the import cap	input-j/time
H	Minimum autonomy or stockout horizon	time
$L_\odot$	Solar luminosity	W
$M_\odot$	Solar mass	mass
G	Gravitational constant	length^3/(mass time^2)
c	Speed of light	length/time
Q_{pr}	Radiation-pressure efficiency	dimensionless
β_{sail}	Radiation-pressure to gravity acceleration ratio	dimensionless
σ_A	Total areal mass density	mass/area
σ_{SB}	Stefan-Boltzmann constant	W/(area temperature^4)
A_R	Radiator planform area	area
n	Effective number of radiating faces	dimensionless

Symbol	Definition	Units or range
ϵ	Radiator emissivity	$[0, 1]$
T_R	Radiator temperature	temperature

13 Appendix B: Variable-growth leverage

For a time-varying net growth rate,

$$\frac{d \ln K}{dt} = g(t)$$

implies

$$K(T) = K(0) \exp\left(\int_0^T g(t)\, dt\right).$$

If no Earth capacity is added after $t = 0$ and all post-seed net gain meets the declared nonterrestrial source rule, net installed leverage is

$$\mathcal{B}_T^{\text{net}} = \frac{K(0)}{K_{\text{seed,Earth}}} \left[\exp\left(\int_0^T g(t)\, dt\right) - 1\right].$$

The familiar expression without the leading ratio assumes $K(0) = K_{\text{seed,Earth}}$. No single arithmetic average of annual percentage changes should be substituted for the integral. A time-weighted log rate preserves the capacity identity.

For gross leverage with time-varying commissioning and seed additions,

$$\mathcal{B}_{[t_0,t_1]}^{\text{gross}} = \frac{\int_{t_0}^{t_1} I_{K,\text{NT}}(t)\, dt}{K_{\text{seed,Earth}}},$$

but continuing physical imports must be reported separately. Converting unlike imported components into one “seed-equivalent” denominator can conceal the very critical dependencies that the closure vector is intended to expose.

14 Appendix C: Regime acceptance template

A regime claim should complete the following template.

Field	Required declaration
Claimed regime	Earth-supported, space-sustaining, self-expanding, or coordination-limited
System boundary	Included locations, assets, services, and excluded external systems
Evidence window	Start, end, dwell duration, and starting inventories

Field	Required declaration
Stress case	Import ceiling, failures, outages, or other counterfactual applied
Reference basket	Versioned definition of one CE and process requirements
Dashboard	$\dot{m}$, delivered P, $\mathbf{C}$, and g with uncertainty
Binding dependency	Track, process, or input class controlling the result
Source attribution	New Earth, recycled Earth-origin, virgin nonterrestrial, recycled nonterrestrial-origin, and inventory changes
Reliability	Failures, repairs, losses, availability, and retirement
Decision	Acceptance criteria met, not met, or not measured

This format makes an honest "not measured" preferable to an unsupported maturity label.

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